

# LOW-ENERGY QCD: CHIRAL COEFFICIENTS, $U_A(1)$ BREAKING AND THE QUARK-QUARK INTERACTION

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## 1 A Path from QCD to Chiral Perturbation Theory

A global color symmetry model (GCM) that is based upon an effective quark-quark interaction arises from the QCD partition function by formally integrating over the gluon fields and truncating the expansion in gluon  $n$  point functions after  $n = 2$ .<sup>1</sup> This model truncation respects all global symmetries of QCD, in particular chiral symmetry. What is lost is invariance under local color gauge transformations. Fixing a model form for the gluon propagator  $D(s)$ , or, what is equivalent, the running coupling  $\alpha(s)$ , specifies a quark-quark interaction. Furthermore the model allows a  $\frac{1}{N_c}$  expansion. The lowest order ( $\mathcal{O}(N_c)$ ) consists in solving the Dyson-Schwinger equation for the quark self energy including rainbow gluon dressings and the ladder Bethe-Salpeter equation for mesonic bound states. One then is able to change the degrees of freedom from quarks to hadrons by performing an appropriate variable transformation in the generating functional path integral. For the moment only the Goldstone bosons are taken into account. We obtain a non-local hadronic interaction

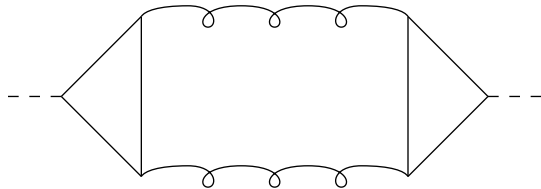
Table 1: The chiral coefficients using a running coupling  $\alpha(s)$ . The parameter choices listed maintain  $f_\pi = 86\text{MeV}$ , which is the generic scale of dynamical chiral symmetry breaking. The numbers in parantheses are the “phenomenological” values.

$\alpha_1(s) = 3\pi s\chi^2 e^{-s/\Delta}/(4\Delta^2) + \pi d/\ln(s/\Lambda^2 + e)$				
$\Delta$ (GeV <sup>2</sup> )	$\chi$ (GeV)	$L_1(0.7\pm 0.5)$	$L_3(-3.6\pm 1.3)$	$L_5(1.4\pm 0.5)$
0.002	1.4	0.84	-4.4	1.0
0.02	1.5	0.82	-4.4	1.14
0.2	1.65	0.81	-4.0	1.66
0.4	1.84	0.80	-3.8	2.0

between the Goldstone fields, which can be systematically expanded in the momenta of the Goldstone fields and the current quark mass leading exactly to the form given by Gasser and Leutwyler.<sup>2</sup> The chiral low energy coefficients  $L_i$  are now determined by the dynamics of the quark-quark interaction and can be numerically calculated from the quark self energy functions.<sup>3</sup> As one can see from Tab.1,  $L_1$  and  $L_3$  are practically independent on the quark-quark interaction, whereas  $L_5$  depends noticeably on its form.

## 2 Triangle Diagrams, $U_A(1)$ Breaking and the $\eta'$ Mass

Including higher mass mesonic states other than the Goldstone fields and integrating them out generates triangle diagrams such as



Those diagrams are suppressed by one order of  $\frac{1}{N_c}$  compared with ones mentioned in the last section. They break the  $U_A(1)$  symmetry and depending on the behavior of the modelled running coupling  $\alpha(s)$  in the infrared can create a mass for the iso-singlet pseudoscalar Goldstone boson  $\eta'$ . This provides a mechanism for an  $\eta'$  mass without employing explicitly topological gauge field configurations (instantons).<sup>4</sup>

## References

1. C.Roberts, R.Cahill and J.Praschifka, *Ann.Phys.(N.Y.)* **188**, 20 (1988).
2. J.Gasser and H.Leutwyler, *Nucl. Phys. B* **250**, 465 (1985).
3. M.Frank and T.Meissner, *Phys. Rev. C* **53**, 2410 (1996).
4. M.Frank and T.Meissner, in preparation.